A Gauge Transformation of Electromagnetic Potentials for Decomposition of Harmonic Fields

Katsumi Yamazaki and Satoshi Kuramochi
Chiba Institute of Technology
2-17-1, Tsudanuma, Narashino, Chiba 275-0016, Japan, yamazaki.katsumi@it-chiba.ac.jp

Abstract — A gauge transformation of electromagnetic potentials is proposed for decomposition of harmonic fields in electric machines. By using the proposed method, the harmonic electromagnetic fields can be obtained only by Fourier transformation of the magnetic vector potential. As a consequence, the required computer memory for the harmonic field decomposition is significantly reduced. In addition, the uniqueness of the decomposed harmonic fields is ensured. The necessity and usefulness of the proposed method is verified by the application to several electric machines.

I. INTRODUCTION

The electromagnetic field in electric machines includes various kinds of time harmonics, for example, slot harmonics and inverter-carrier harmonics. Therefore, it is important for the machine design to separately estimate the harmonic fields.

The time-harmonic electromagnetic field distributions can be obtained by using Fourier transformation of the flux and current density obtained by the time-stepping finite element analysis. This method is well known and widely applied in the 2-D analysis. However, in the case of the 3-D analysis, large computer memory and calculation time will be required because both of the 3-D flux and eddy current vectors at each time step should be memorized and the Fourier transformation of each component should be carried out at each 3-D finite element. This must be one reason why the 3-D harmonic field decomposition is rarely reported, even though it is useful for the design of electric machines.

From these viewpoints, we proposed a method for harmonic field decomposition by using the gauge transformation of electromagnetic potentials in a post process. In the method, only 3-components of the magnetic vector potential (A_x, A_y, A_z) are required for the harmonic electromagnetic field calculation. In addition, the uniqueness of the decomposed harmonic fields is ensured As a result, the required memory and calculation time for the harmonic decomposition is significantly reduced as compared to the conventional method.

The proposed method is applied to several kinds of electric machines to verify the necessity and usefulness.

II. CALCULATION METHOD

A. Equations of electromagnetic field

In the case of the edge-finite element method, the electromagnetic field can be represented by following equation by using the magnetic vector potential $\boldsymbol{A}$*:

$$\nabla \times (\frac{1}{\mu} \nabla \times \boldsymbol{A}^*) = \boldsymbol{J}_c - \sigma \frac{\partial \boldsymbol{A}^*}{\partial t} + \frac{1}{\mu_0} \nabla \times \boldsymbol{M} \quad (1)$$

where μ and σ are the permeability and the conductivity, respectively; $\boldsymbol{J}_c$ is the winding current density; $\boldsymbol{M}$ is the magnetization of the permanent magnet.

Alternatively, the field can also be represented by the magnetic vector potential $\boldsymbol{A}$ and electric potential ϕ as:

$$\nabla \times (\frac{1}{\mu} \nabla \times \boldsymbol{A}) = \boldsymbol{J}_c - \sigma \left(\frac{\partial \boldsymbol{A}}{\partial t} + \nabla \phi \right) + \frac{1}{\mu_0} \nabla \times \boldsymbol{M} \quad (2)$$

$$\nabla \bullet \left\{ \sigma \left(\frac{\partial \boldsymbol{A}}{\partial t} + \nabla \phi \right) \right\} = 0 \quad (3)$$

This formulation has two advantages over (1) in the analyses of electric machines. One is the quick convergence of the matrix solver [1]. Although it requires both $\boldsymbol{A}$ and ϕ, the calculation time to solve the matrix is relatively short because of the stable convergence in ICCG method. The other is the easy modeling for insulation of electrical steel sheets and divided magnets in the electric machines [2], [3]. The finite element subdivision within the thin insulated layer can be avoided by setting ϕ at the insulated surface.

On the other hand, the potentials $\boldsymbol{A}$ and ϕ are not unique. Only the uniqueness of electromagnetic field is ensured. This will cause one problem in the harmonic field decomposition, as discussed in the next section.

B. Decomposition of harmonic electromagnetic fields

From the solutions of (2) and (3), the flux and eddy current densities can be calculated as follows:

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \quad (4)$$

$$\boldsymbol{J} = \sigma \left(-\frac{\partial \boldsymbol{A}}{\partial t} - \nabla \phi \right). \quad (5)$$

By applying the Fourier transformation to the time-series data of (4) and (5), $\boldsymbol{B}$ and $\boldsymbol{J}$ can be decomposed into harmonic components, as follows:

$$\boldsymbol{B} = \sum_n \boldsymbol{B}_n \cos(n\omega t + \theta_{Bn}) \quad (6)$$

$$\boldsymbol{J} = \sum_n \boldsymbol{J}_n \cos(n\omega t + \theta_{Jn}) \quad (7)$$

where n is the time harmonic order; ω is the fundamental angular frequency; $\boldsymbol{B}_n$ and $\boldsymbol{J}_n$ are the amplitudes of the n^{th} harmonics; θ_{Bn} and θ_{Jn} are the phases of the harmonics. However, these decompositions require large computer

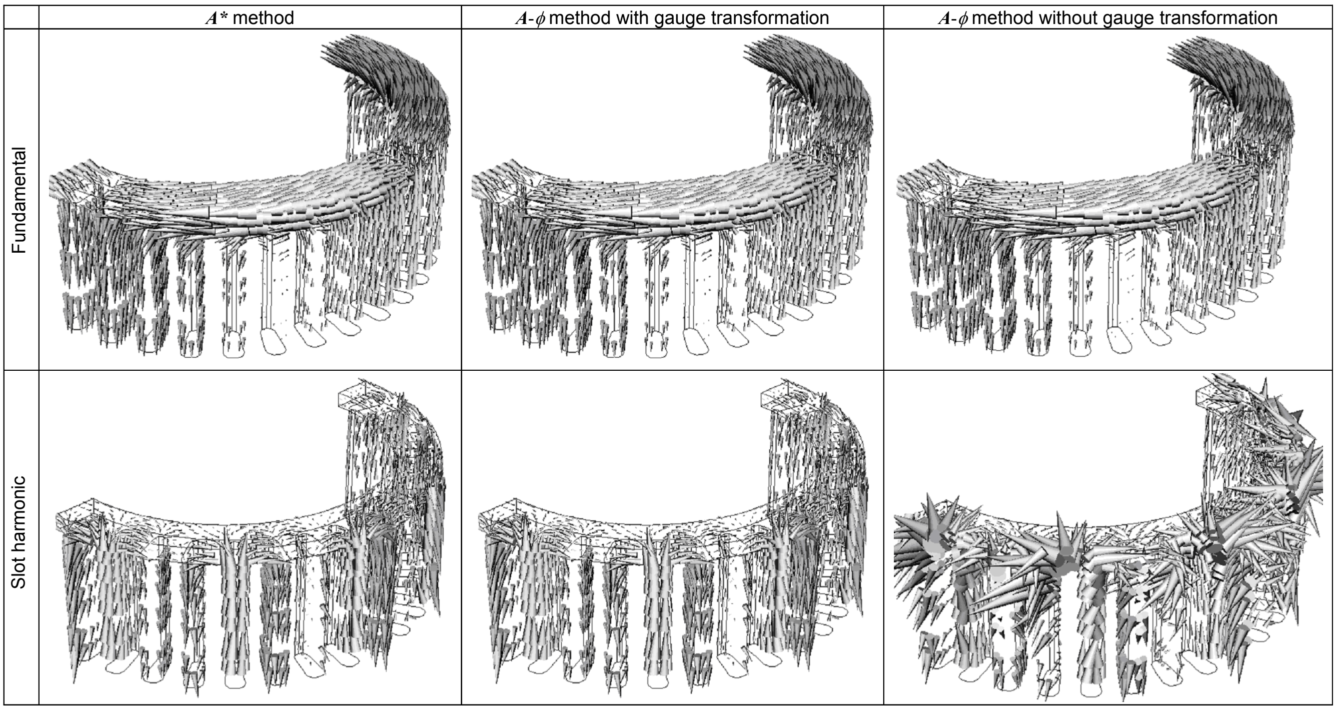

Fig.1. Decomposed harmonic rotor cage currents in induction motor (V_1=100V, f=50Hz, slip=0.33).

memory and calculation time because the time-series data of 6-components (B_x, B_y, B_z, J_x, J_y, J_z) at each finite elements in the conductive region should be memorized.

On the other hand, the electromagnetic potentials $\boldsymbol{A}$ and ϕ can also be decomposed into harmonic potentials as:

$$\boldsymbol{A} = \sum_n \boldsymbol{A}_n \cos(n\omega t + \theta_{An}) \tag{8}$$

$$\phi = \sum_n \phi_n \cos(n\omega t + \theta_{\phi n}) \tag{9}$$

From (4)-(9), $\boldsymbol{B}_n$ and $\boldsymbol{J}_n$ can be expressed, as follows:

$$\boldsymbol{B}_n = \nabla \times \left\{ \boldsymbol{A}_n \cos(n\omega t + \theta_{An}) \right\} \tag{10}$$

$$\boldsymbol{J}_n = n\omega\sigma \boldsymbol{A}_n \sin(n\omega t + \theta_{An}) - \sigma \nabla \phi_n \cos(n\omega t + \theta_{\phi n}) \tag{11}$$

In this case, the harmonic electromagnetic field can be calculated from the time-series data of (A_x, A_y, A_z, ϕ). However, as discussed in the previous section, the potentials $\boldsymbol{A}$ and ϕ are not unique. As a consequence, the definitions of $\boldsymbol{A}$ and ϕ at each time step must be different. In this case, the harmonic decomposition due to (10) and (11) will cause unphysical results because Fourier transformation of $\boldsymbol{A}$ and ϕ has no meanings.

Therefore, we introduce following gauge transformation for $\boldsymbol{A}$ and ϕ before the harmonic decomposition:

$$\boldsymbol{A}^* = \boldsymbol{A} + \int_0^t \nabla \phi dt \tag{12}$$

This expression means the transformation from the potentials of $\boldsymbol{A}$-ϕ method to $\boldsymbol{A}$* method, whose potential is unique. In this case, $\boldsymbol{B}_n$ and $\boldsymbol{J}_n$ can be expressed, as follows:

$$\boldsymbol{B}_n = \nabla \times \left\{ \boldsymbol{A}_n{}^* \cos(n\omega t + \theta_{An}) \right\} \tag{13}$$

$$\boldsymbol{J}_n = n\omega\sigma \boldsymbol{A}_n{}^* \sin(n\omega t + \theta_{An}) \tag{14}$$

where $\boldsymbol{A}_n$* is the harmonic magnetic vector potential decomposed by the Fourier transformation with the gauge of ϕ=0. By using this method, the uniqueness of the harmonic potential is ensured. In addition, the further reduction of computer memory is achieved because only the time-series data of (A_x, A_y, A_z) is required in this method.

III. Results and Discussion

Fig. 1 shows the decomposed harmonic rotor cage currents of an induction motor. The currents are generated by the fundamental and stator-slot harmonic fields. The fundamental currents flow in a one pole-pitch loop, whereas the slot harmonic currents flow in short pitch loops. The result of the $\boldsymbol{A}$* and $\boldsymbol{A}$-ϕ method with the gauge transformation are almost identical. However, the calculation time to solve the matrix of the $\boldsymbol{A}$-ϕ method is shorter than that of the $\boldsymbol{A}$* method due to the stable convergence of the ICCG method. It is also observed that the slot harmonic component obtained by the $\boldsymbol{A}$-ϕ method without the gauge condition shows unphysical distribution because the uniqueness of the harmonic electromagnetic potentials is not ensured. The necessity and usefulness of the proposed method is verified. Further results and discussion will be shown in the extended paper.

IV. References

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